Modified Newton Filters for Edge Orientation Estimation and Shape Representation

M. Alemán-Flores, L. Álvarez-León and R. Moreno-Díaz jr.
Departamento de Informática y Sistemas
Universidad de Las Palmas de Gran Canaria
Campus de Tafira, 35017 Las Palmas, SPAIN
{maleman, lalvarez, rmorenoj}@dis.ulpgc.es

Abstract

The identification of an object is provided, in most cases, by a description of its shape, which requires an accurate location of its borders. However, it is important that changes in orientation, size or contrast between the object and the background do not alter the output of the mechanism used for the recognition. In this article, we present a set of formal tools, based on Newton filters, which allow estimating edge orientation and whose output does not vary when the input signal is rotated or when a global illumination change occurs. Furthermore, the operations are performed in a layered structure which simulates retinal computations. The outputs of these filters are used to build a one-dimensional representation of the contour in order to characterize shapes by using Fourier coefficients.

Keywords: Filter Design, Edge Orientation, Shape Representation, Fourier Transform.

1 Introduction

This article presents a new set of tools, the modified Newton filters, and their application to shape representation. Many sets of filters can be found in the literature to locate the edges of and object. Among them, we can find Newton filters [Mor93]. However, our goal is not the detection itself, but the characterization of edges according to their orientation, since this provides much more information on the shape that is presented and allows a further analysis of the objects. From Newton filters, which have been previously used as neuron-like structures for retinal processing [QAM99], a new set is built with some advantageous features. We show the way these filters are obtained and their use for estimating edge orientation.

The outputs which these filters provide in every point along the contour of an object allow building a one-dimensional representation of its shape, in such a way that we can compare patterns and classify objects according to their outlines. To do this, a Fourier analysis is carried out on the one-dimensional functions which have been generated. In order to generalize our classifying method as much as possible, we have considered the consequences of certain transformations on the objects as well as the different conditions in which a given shape can be found. By studying how these transformations alter the coefficients, we have determined the mechanism to compare and match the contours.

2 Modified Newton filters

Newton filters are tools based on the repeated use of simple binary operations to build more complex ones, which can compute a wider range of inputs. If we use addition and subtraction of two real or integer numbers as basic operations, we can combine them in different layers in such a way that the resulting functions are linear combinations of the inputs. Each layer operates on the ordered set of results of the previous layer, and the operation which is carried out into a layer is the same for each unit inside it, regardless of the position on which it is performed. Nevertheless, the operations of different layers may vary. The mathematical properties of this kind of filters were deeply studied in [Mor93].

We have developed a set of filters which preserve the convenient properties of original Newton filters, but which also avoid some of the undesirable phenomena by providing them with rotational invariance and non-null weights in all positions. Firstly, as we are going to work with 3x3 filters, we calculate the one-dimensional filter of length 3 that contains only additive layers $NF(A_2, D_0) = (1, 2, 1)$. From this values, we build filters that react to changes in the 8 main orientations.

$$\underset{F_0\,:\,0}{\begin{bmatrix} 1 & 1 & -2 \\ 2 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}} \quad \underset{F_2\,:\,\pi/2}{\begin{bmatrix} -2 & -4 & -2 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}} \quad \underset{F_4\,:\,\pi}{\begin{bmatrix} -2 & 1 & 1 \\ -4 & 2 & 2 \\ -2 & 1 & 1 \end{bmatrix}} \quad \underset{F_6\,:\,3\pi/2}{\begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ -2 & -4 & -2 \end{bmatrix}}$$

$$\underset{F_1\,:\,\pi/4}{\begin{bmatrix} 1 & -2 & -4 \\ 1 & 2 & -2 \\ 2 & 1 & 1 \end{bmatrix}} \quad \underset{F_3\,:\,3\pi/4}{\begin{bmatrix} -4 & -2 & 1 \\ -2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}} \quad \underset{F_5\,:\,5\pi/4}{\begin{bmatrix} 1 & 1 & 2 \\ -2 & 2 & 1 \\ -4 & -2 & 1 \end{bmatrix}} \quad \underset{F_7\,:\,7\pi/4}{\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & -4 \end{bmatrix}}$$

Table 1: Modified Newton filters and corresponding orientation.

While in the original Newton filters we observed differences according to the orientation of the edge we wanted to detect, in this set, the weights are arranged cyclically and rotations of the image do not alter the magnitude of the output, but only the indices. A more complete information about the orientation of the edges is given by considering the whole pattern provided by the eight filters. Below we can see the output of these eight filters for a $\pi/2$ oriented edge.

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	5	8	5	0	−4	−4	−4

If we change the orientation with a $\pi/4$ step, the output is only cyclically shifted, but the values and their order are not altered. Thus, by correlating the ideal pattern with the real one, we can determine how perfect the border is as well as how it is oriented. The value

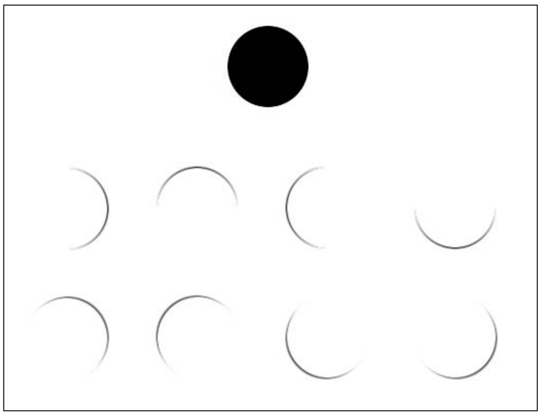

Figure 1: Output of the modified Newton filters for the circle above.

of x for which this correlation is maximum determines which orientation is more similar to the real one. As the change from one side of the border to the other one may be larger than 1, it is necessary to normalize the output. When the real orientation does not correspond to one of these directions, we can estimate it by interpolating the higher value with its two neighbors, which provides an accurate estimation of the edges. Thus, the main advantage of this kind of filters is not the location of edges, but their classification according to their orientation and the invariance under rotations and illumination changes.

3 Contour-based shape representation

Once we have located the edges and the orientation in every point of the contour, we can build a one-dimensional sequence which describes how the latter changes as we move along the border. If the edges are not clear enough, double borders may appear and must be eliminated before continuing.

We must first be placed on a point of the outline and, according to its orientation, find the neighbor that best matches the orientation it indicates. If we continue searching for non-visited neighbors until we close the figure, at the end of the process we must have covered the border of the object and found a description of its orientation function.

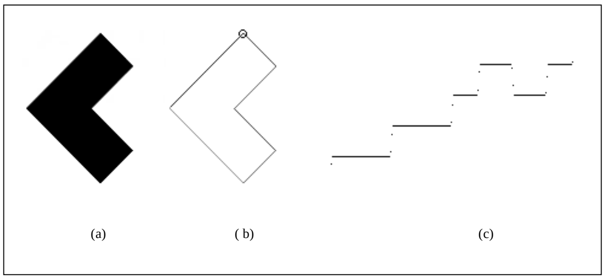

Figure 2: (a) Input image. (b) Outline with orientation values -starting point for orientation function signalled-. (c) Orientation function.

The values of the orientation functions are selected in such a way that no point differs in more than π radians from its neighbors, except for the first and the last points of a closed curve, which are actually neighbors. We have reduced the problem of identifying shapes, i.e. two-dimensional functions, to the association of two one-dimensional functions. However, no condition has been set on the starting point, as having the whole object in the image generates a closed curve.

The way the orientation function is obtained makes the starting point very significant. In fact, depending on the point of the contour we select to start, a different result may be generated. Nevertheless, these results will only differ in a certain shift and a constant, 2π, which is added to those values which appear on the other side of the signal when it is circularly shifted. If we wish to identify a pattern regardless of the first point we select, we must previously shift the resulting function to be able to compare it with the reference pattern and study how such constant affects the final result. In order to extract the appropriate shift, we use Fourier coefficients. These coefficients will provide not only the best shift to compare the signals, but also a similarity function to determine how well they match. In [ZR72], the optimality criteria, as well as the consequences of certain kinds of transformations on Fourier coefficients are studied. However, the type of signals and the discrimination function which are used differ from those we deal with.

We are interested in identifying shapes regardless of their sizes. Hence, the sequences obtained for orientation functions are normalized in their length, in such a way that all of them are equally long. Moreover, the length we use is always a power of 2, in order to use fast Fourier transform and reduce the computational cost.

When the object is rotated, all points on the contour will undergo an increase in the values of their respective orientations. However, this increase will be the same for all of them, provided it is a solid whose shape is not altered by the rotation. This phenomenon only affects order zero coefficient of the orientation function. Let g_n be the orientation signature of the object described by f_n after a rotation of a certain angle θ, its coefficients are the following:

$$\widehat{g}_k = \frac{1}{L}\sum_{n=0}^{L-1} g_n e^{-i\frac{2\pi kn}{L}} = \frac{1}{L}\sum_{n=0}^{L-1}(f_n+\theta)e^{-i\frac{2\pi kn}{L}} = \begin{cases} \theta + \frac{1}{L}\sum_{n=0}^{L-1} f_n = \widehat{f}_0 + \theta & if \quad k=0 \\ \frac{1}{L}\sum_{n=0}^{L-1} f_n e^{-i\frac{2\pi kn}{L}} = \widehat{f}_k & if \quad k \neq 0 \end{cases}$$

As said before, if we do not force the algorithm to start in the same point of the figure, not only all the values are shifted, but also some of them are increased in 2π. Let g_n be the signal resulting of shifting f_n a positions and increasing the values which appear in the opposite side in 2π, its coefficients are the following:

$$\widehat{g}_k = \frac{1}{L}\sum_{n=0}^{L-1} g_n e^{-i\frac{2\pi kn}{L}} = e^{i\frac{2\pi ka}{L}}\widehat{f}_k + \frac{2\pi\left(e^{i\frac{2\pi ka}{L}}-1\right)}{L\left(1-e^{-i\frac{2\pi k}{L}}\right)} \quad \forall\ k \neq 0 \tag{1}$$

4 Energy function for shape characterization

We can use any but order 0 coefficient, since it is related to the mean value, and it should not affect the result. Moreover, the higher the order of the coefficient, the more sensitive it is to noise. However, when a shape has r-fold rotational symmetry and fits itself under a rotation of $2\pi/r$, those coefficients which are not multiples of r are null, thus avoiding to extract a right shift from them. We can build an energy function which adds the errors for every coefficient. From equation (1), we can determine how good the relationship between the coefficients is for a certain value of a and a given order k:

$$E_k(a) = e^{i\frac{2\pi ka}{L}}\left(\widehat{f}_k + \frac{2\pi}{L\left(1-e^{-i\frac{2\pi k}{L}}\right)}\right) - \left(\widehat{g}_k + \frac{2\pi}{L\left(1-e^{-i\frac{2\pi k}{L}}\right)}\right) = e^{i\frac{2\pi ka}{L}}\widetilde{f}_k - \widetilde{g}_k$$

If we add the terms corresponding to every coefficient with non-null index, multiplying each one of them by its respective conjugate, we obtain the function in equation (2), where $\widetilde{f}_k$ and $\widetilde{g}_k$ are obtained from Fourier coefficients as shown above. Taking into account that the higher the order of the coefficient, the more sensitive it is to noise, the energy factors have been weighted in such a way that the first coefficients are given a higher weight than the last ones.

$$W(a) = \sum_{k=1}^{\frac{L}{2}} \frac{L-k+1}{L}\left(\left|\widetilde{f}_k\right|^2 + \left|\widetilde{g}_k\right|^2 - e^{i\frac{2\pi ka}{L}}\widetilde{f}_k\widetilde{g}_k^* - \left(e^{i\frac{2\pi ka}{L}}\widetilde{f}_k\widetilde{g}_k^*\right)^*\right) \tag{2}$$

With this new expression, those alterations of the signal due to the presence of noise, which affect more strongly higher order coefficients, are not so significant. Table 2 shows the final values of normalized minimum energy for 9 images corresponding to 3 different keys (see figure 3) when the factors are weighted as in equation (2). In this case, $kn : m$ corresponds to the m^{th} image of the n^{th} key (n^{th} key of the m^{th} row). The normalization which has been carried out consists in dividing the values by the mean energy obtained when comparing two different images of the same key.

Figure 3: Images of three different keys in different positions and showing both sides.

$W_{\min}$	$k1:1$	$k1:2$	$k1:3$	$k2:1$	$k2:2$	$k2:3$	$k3:1$	$k3:2$	$k3:3$
$k1:1$	0.0000	1.1385	0.8397	3.7433	4.3798	4.3227	7.2674	6.3000	5.9164
$k1:2$		0.0000	0.5509	5.6977	6.1319	6.0507	8.0686	6.8423	6.6339
$k1:3$			0.0000	4.8914	5.5298	5.3115	7.9518	6.5084	6.3362
$k2:1$				0.0000	0.8658	1.1344	9.3063	9.2261	8.5619
$k2:2$					0.0000	1.1442	9.6234	9.5260	7.0714
$k2:3$						0.0000	9.5537	9.4011	8.9901
$k3:1$							0.0000	1.3674	1.1965
$k3:2$								0.0000	0.7626
$k3:3$									0.0000

Table 2: Normalized minimum weighted energy values for keys in figure 3.

We have already considered the problem of starting the orientation function of a closed curve in a different point, but next, we must see what happens if we choose the opposite direction to continue. If we go through the contour starting at the same point as in the reference pattern, the sequence will be transformed. Nevertheless, this phenomenon can be detected as, for closed curves, the difference between the first and the last points of the sequence will be -2π if we progress clockwise and 2π if we do it counterclockwise. If we are working with plane objects, e.g. keys, they can be presented in two different forms, corresponding to both sides of the object. However, if g_n is a reflected version of f_n, the orientation functions can be coupled if we consider the changes they undergo. If we start at the same point, the sequence and its Fourier transform coefficients are transformed as follows:

$$\widehat{g}_k = \frac{1}{L}\left(C - f_0 + \sum_{n=1}^{L-1}\left(C - f_{L-n} + 2\pi\right)e^{-i\frac{2\pi k n}{L}}\right) = \begin{cases} -\widehat{f}_0 + \frac{2\pi(L-1)}{L} + C & if \quad k = 0 \\ -\widehat{f}_{-k} - \frac{2\pi}{L} & if \quad k \neq 0 \end{cases}$$

where C is a value which depends on the symmetry axis that has been used for reflection and the starting point of the contour, but which remains constant for all points inside the sequence. From this expression, a relationship can be extracted to test whether both contours correspond to the same or different sides (last row of figure 3).

5 Conclusions

The present article proposes new tools for calculating and identifying edges which are inspired on Newton Filters. Some modifications have been carried out on this set of filters in order to provide them with rotation invariance, but preserving their invariance to global illumination changes. Similar filters have been proposed by Prewitt, Sobel, Robinson or Kirsch [SHB99], but they were not extracted from natural simulation, cause the duplication of edges, are independent of the central value or may produce the maximum output for imperfect edges. We not only locate the edges, but also identify their orientation in an accurate way, which is very important when studying curvature, singular points or selective motion detection of an object.

From these kind of basic tools, it is easy to build orientation functions which can clearly identify shapes and extract global information to fit objects into described patterns. The use of Fourier coefficients provides robust results and allows reducing the computational cost. A similar mechanism for shape identification, based on continuous Fourier series, is used in [ZR72], but oriented to polygonal shapes with irregularly spaced series of points, while in our case, points are equally separated, and we use the fast Fourier transform as basic tool for the shape analysis.

References

[Mor93] Moreno-Díaz R.jr.: *Computación paralela y distribuida: relación estructura-función en retinas.* Tesis Doctoral, 1993.

[QAM99] Quesada-Arencibia A., Alemán-Flores M. and Moreno-Díaz R.jr.: *Newton Filters: a new class of neuron-like discrete filters and an application to image processing.* Artificial Neural Nets and Genetic Algorithms, pp. 28-34. Springer Computer Science, 1999.

[SHB99] Sonka M., Hlavac V. and Boyle R.: *Image Processing, Analysis, and Machine Vision.* PWS-ITP, 1999.

[ZR72] Zahn C. and Roskies R.: *Fourier descriptors for plane closed curves.* Computer Graphics and Image Processing, 21, pp. 269-281, 1972.