3D Motion Estimation using a Combination of Correlation and Variational Methods for PIV

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1 Introduction

Lots of work has been carried out using Particle Image Velocimetry to design experiments which capture and measure the flow motion using 2D images. Recent technological advances allow capturing 3D PIV image sequences of moving particles. In this context, we propose a 3D motion estimation technique based on the combination of an iterative cross-correlation technique and a variational (energy-based) technique. A combination of both methods (using the output of the correlation technique as the initial input of the variational method) improves the accuracy of the flow estimation.

2 Local Cross-Correlation

Cross-correlation is the most common technique for fluid motion estimation in PIV and is described in [2]. Having the two volumes I_1 and I_2, for each voxel $\mathbf{v} = (v_x, v_y, v_z)$ of I_1, the method takes a rectangular subvolume $I_{1,\mathbf{v}}$ of I_1 centered on $\mathbf{v}$, and looks for a similar subvolume of I_2 centered on a neighbor $\mathbf{v} + \mathbf{d}$ of $\mathbf{v}$. The similarity measure between two rectangular subvolumes of the same dimensions is based on 2D cross-correlation. The voxel $\mathbf{v}$ is assigned the displacement $\mathbf{d}$ which gives the maximal value of the cross-correlation. Doing this for every voxel in I_1 we obtain a complete motion vector field $\mathbf{u}$. The method is then extended to allow subvoxel accuracy by means of local interpolation of a Gaussian function close to the discrete maximum.

The implementation takes advantage of the properties of the Fourier transform to improve the processing time and the whole process should be applied iteratively a few times using the current result as an initialization for the next iteration.

3 Variational Approach

Variational approach to motion estimation are often used for optical flow computation [1]. It consists in minimizing an energy as a function of the displacement and that depends on a pair of images I_1 and I_2.

The energy to minimize is expressed as :

$$E(\mathbf{u}) = \underbrace{\int_{\Omega} \left(I_1(\mathbf{x}) - I_2\left(\mathbf{x} + \mathbf{u}(\mathbf{x})\right)\right)^2 d\mathbf{x}}_{data\ term} + \underbrace{\alpha \int_{\Omega} \|\nabla \mathbf{u}(\mathbf{x})\|^2 d\mathbf{x}}_{regularization\ term}, \tag{1}$$

where α is a scalar coefficient that weights the smoothing term.

We use an iterative method to find the vector field $\mathbf{u}$, updating the vector field at each iteration by adding another vector field $\mathbf{h}$ with small displacements. The displacement $\mathbf{h}$ being small, we can use first order Taylor expansions of I_2 and ∇I_2 at $\mathbf{x} + \mathbf{u}^n$ to linearize the minimization problem. Furthermore, we use a pyramidal approach to compute the displacement flow at several scales, using the results from a given scale to initialize to the following higher scale.

4 Results

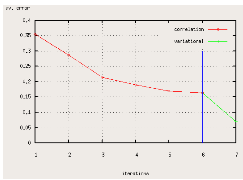

Fig. 1. Average error evolution using the combined scheme.

For the experiments we used synthetic data and 3D flows based on realistic flow models to check the performance of the proposed methods. In these experiments, we first apply the correlation method to obtain a good approximation of the flow and then we refine the results with the variational approach.

Figure 1 shows that the correlation reaches a stable average error after 6 iterations and that an additional iteration of the variational approach reduces it even more.

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References

1. S. S. Beauchemin and J. L. Barron. The computation of optical flow. *ACM Computing Surveys*, 27(3):433–467, 1995.
2. M. Raffel, C. Willert, and J. Kompenhans. *Particle Image Velocimetry. A Practical Guide*. Springer Verlag, 1998.